Quantum Mechanics

Sample exam 5

Christophe De Beule (christophe.debeule@gmail.com)

Question 1 – Perturbation theory

Consider a particle in a harmonic oscillator potential with a time-dependent angular frequency $\omega(t) = \omega_0 + \varepsilon(t)$. The Hamiltonian is

$$\begin{aligned} H(t) &= \frac{p^2}{2m} + \frac{1}{2}m\left(\omega_0 + \varepsilon(t)\right)^2 x^2 \\ &\simeq \underbrace{\frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 x^2}_{H_0} + \underbrace{m\omega_0\varepsilon(t)x^2}_{V(t)} . \end{aligned}$$

(a) Assume that the particle is initially in the ground state of H_0 at time t_0 and find a general expression for the first-order amplitudes $c_n^{(1)}(t)$.

(b) Calculate the coefficients for $t_0 = 0$ and $\varepsilon(t) = \varepsilon_0 \sin \Omega t$. Write down $|\psi(t)\rangle$ formally in lowest order and find the expectation value of x^2 in lowest order.

(c) Now consider $\varepsilon(t) = \frac{\varepsilon_0}{\sqrt{\pi}} e^{-t^2/\tau^2}$ and $t_0 \to -\infty$. Find the transition probability at $t \to \infty$. What happens to the system for $\omega_0 \tau \gg 1$ (adiabatic limit)?

Question 2 – Scattering theory

Consider scattering of a particle of mass m at a hard sphere with radius a and a delta shell at radius $b > a$:

$$V(r) = \begin{cases} \infty & r \leq a \\ \lambda\delta(r-b) & r > a. \end{cases}$$

(a) Write down the general solution for s-wave scattering ($l = 0$) in each region and the boundary conditions at $r = a$ and $r = b$.

(b) Find an expression for the s-wave S-matrix element $S_0 = e^{2i\delta_0}$ where $\delta_0(k)$ is the s-wave phase shift.

(c) Discuss the limits $a \to 0$, $b \to a$, $m\lambda/\hbar^2 \to 0$, and $m\lambda/\hbar^2 \to \infty$.

Question 3 – Relativistic quantum mechanics

Find an expression for the s-wave ($l = 0$) bound states of a relativistic *spinless* particle inside a spherical potential well:

$$V(r) = \begin{cases} -V_0 & r \leq a \\ 0 & r > a, \end{cases}$$

where $V_0 > 0$. Introduce the potential with the substitution $E \to E - V(r)$ and consider only solutions in the energy range $|E| < mc^2$ and $|E + V_0| > mc^2$.